**1.** A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

**(1)**

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.

**(3)**

(c) Comment on the suitability of a binomial distribution to model this situation.

**(1)**

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

**(4)**

(e) Find the *p*-value in this case.

**(1)**

a) let $R$ be the number of red beads in Aliya's bracelet

$R \sim B(18, 0.14)$ 1

b) i) $P(R=1) = 0.19403... = 0.194$ (3sf) 1

ii) $P(R \geq 4) = 1 - P(R \leq 3) = 1 - 0.76184...$ 1 $= 0.2381...$
$= 0.238$ (3sf) 1

c) a condition for a binomial distribution is that $p$ is constant. so there must be a large number of beads in the bag such that removing 18 doesnt significantly change $p$. 1

two tailed ∴ halve significance level

d) $H_0: p = 0.14$ $H_1: p \neq 0.14$ (1)

Assume $H_0$ correct
let $X$ = # of red beads in the sample

sig. level $= 2.5\% = 0.025$

$X \sim B(75, 0.14)$ (1)

$P(X \leq 4) = 0.01506... < 0.025$ (1)

- Sufficient evidence to reject $H_0$ (1)
- There is evidence to suggest that the proportion of red beads was changed. (1)

two tailed so don't say increased/decreased.

e) p-value is $2 \times 0.01506... = 0.0301...$
$= 0.030$ (2sf) (1)

**2.** Past information shows that 25% of adults in a large population have a particular allergy.

Rylan believes that the proportion that has the allergy differs from 25%

He takes a random sample of 50 adults from the population. two tailed

Rylan carries out a test of the null hypothesis $H_0$: $p = 0.25$ using a 5% level of significance.

(a) Write down the alternative hypothesis for Rylan's test.

**(1)**

(b) Find the critical region for this test.
You should state the probability associated with each tail, which should be as close to 2.5% as possible.

**(4)**

(c) State the actual probability of incorrectly rejecting $H_0$ for this test.

**(1)**

Rylan finds that 10 of the adults in his sample have the allergy.

(d) State the conclusion of Rylan's hypothesis test.

**(1)**

a) $H_1: p \neq 0.25$ (1)

b) let $X$ be the number of adults with the allergy.
Assume $H_0$ correct: $X \sim B(50, 0.25)$ (1)

significance level for each tail = 2.5% = 0.025

$P(X \leq 5) = 0.00705$
$P(X \leq 6) = 0.01939$ ← closest to 0.025
$P(X \leq 7) = 0.04526$ (1)

$P(X \geq 18) = 1 - P(X \leq 17) = 0.05512$
$P(X \geq 19) = 1 - P(X \leq 18) = 0.02873$ ← closest to 0.025
$P(X \geq 20) = 1 - P(X \leq 19) = 0.01392$ (1)

Critical region: $X \leq 6$ and $X \geq 19$ (1)

c) $0.0194 + 0.0287 = 0.048$ (2sf) (1)

d) 10 is not in the critical region ⇒ insufficient evidence to reject $H_0$

There is insufficient evidence to suggest that the proportion of adults with the allergy is different from 25% (1)

link to context!

**3.** A dentist knows from past records that 10% of customers arrive late for their appointment.

A new manager believes that there has been a change in the proportion of customers who arrive late for their appointment.

two tailed

A random sample of 50 of the dentist's customers is taken.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of customers who arrive late
- an alternative hypothesis corresponding to the manager's belief

**(1)**

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis in (a)
You should state the probability of rejection in each tail, which should be less than 0.025

**(3)**

(c) Find the actual level of significance of the test based on your critical region from part (b)

**(1)**

The manager observes that 15 of the 50 customers arrived late for their appointment.

(d) With reference to part (b), comment on the manager's belief.

**(1)**

a) $H_0: p = 0.1 \quad H_1: p \neq 0.1$ (1)

b) Let $X$ be the number of late customers.
Assume $H_0$ correct: $X \sim B(50, 0.1)$ (1)

$P(X \leq 1) = 0.0338 > 0.025$
$P(X \leq 0) = 0.00515 < 0.025$ ←

$P(X \geq 9) = 1 - P(X \leq 8) = 0.0579 > 0.025$
$P(X \geq 10) = 1 - P(X \leq 9) = 0.0245 < 0.025$ ←

CR: $X = 0$ and $X \geq 10$ (1)

$P(X = 0) = 0.00515 \quad P(X \geq 10) = 0.0245$ (1)

c) level of significance $= 0.00515 + 0.0245$
$= 0.0297$ (3sf) 1

d) 15 is in the critical region, so there is evidence to support the manager's belief. 1

**4.** A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.
The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable $T$ represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for $T$ to be modelled by $B(40, \frac{1}{7})$

**(1)**

A box is selected at random.

(b) Using $T \sim B(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

**(2)**

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

**(2)**

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

**(4)**

a) The probability of a packet containing a prize is constant. 1

b) $T \sim B(40, \frac{1}{7})$

(i) $P(T=6) = 0.1727\ldots = 0.173$ (3 s.f.) 1

(ii) $P(T<3) = P(T \leq 2)$

$= 0.061587\ldots = 0.0616$ (3 s.f.) 1

(c) Let r.v. $K$ = number of boxes with fewer than 3 packets containing a prize.

$K \sim B(5, 0.0615...)$ (1)

$\therefore P(K=2) = 0.031344... = 0.0313$ (3 s.f.) (1)

d) Let r.v. $X$ = number of packets containing a prize.

$X \sim B = (110, \frac{1}{7})$ (1)

$H_0 : p = \frac{1}{7}$ , $H_1 = p < \frac{1}{7}$ (1)

$P(x \leq 9) = 0.038292...$ (which is $< 0.05$) (1)

$\therefore$ reject $H_0$ since there is evidence to support Kamil's claim. (1)